**Main Assumptions of Linear Regression Models:**

1. 𝗟 - 𝗟𝗶𝗻𝗲𝗮𝗿𝗶𝘁𝘆: The relationship between the independent and dependent variables should be linear.  
   If violated: The model may underperform, leading to biased predictions.
2. 𝗛 - 𝗛𝗼𝗺𝗼𝘀𝗰𝗲𝗱𝗮𝘀𝘁𝗶𝗰𝗶𝘁𝘆: The variance of residuals should be constant across all levels of the independent variable.  
   If violated: Leads to heteroscedasticity, making standard errors unreliable.
3. 𝗠 - 𝗠𝘂𝗹𝘁𝗶𝗰𝗼𝗹𝗹𝗶𝗻𝗲𝗮𝗿𝗶𝘁𝘆 (No Perfect Multicollinearity): Independent variables should not be highly correlated.  
   If violated: Model coefficients become unstable, affecting interpretation.
4. 𝗔 - 𝗔𝘂𝘁𝗼𝗰𝗼𝗿𝗿𝗲𝗹𝗮𝘁𝗶𝗼𝗻 (No Serial Correlation): Residuals should not be correlated.  
   If violated: Especially in time-series models, it can lead to inefficient predictions and misleading inferences.
5. 𝗡 - 𝗡𝗼𝗿𝗺𝗮𝗹𝗶𝘁𝘆 𝗼𝗳 𝗥𝗲𝘀𝗶𝗱𝘂𝗮𝗹𝘀: Residuals should be normally distributed (essential for small samples).  
   If violated: Hypothesis testing and confidence intervals may become unreliable.
6. Call**:**
7. garch**(**x **=** body\_mass\_change, order **=** c**(**1, 1**)**, trace **=** **FALSE)**
8. Model**:**
9. GARCH**(**1,1**)**
10. Residuals**:**
11. Min 1Q Median 3Q Max
12. **-**2.22070 **-**0.58309 0.05346 0.71924 2.90127
13. Coefficient**(**s**):**
14. Estimate Std. Error t value Pr**(>|**t**|)**
15. a0 0.02921 0.01466 1.992 0.0464 **\***
16. a1 0.20315 0.10562 1.923 0.0544 .
17. b1 0.44416 0.22592 1.966 0.0493 **\***
18. **---**
19. Signif. codes**:** 0 ‘**\*\*\***’ 0.001 ‘**\*\***’ 0.01 ‘**\***’ 0.05 ‘.’ 0.1 ‘ ’ 1
20. Diagnostic Tests**:**
21. Jarque Bera Test
22. data**:** Residuals
23. X**-**squared **=** 0.88359, df **=** 2, p**-**value **=** 0.6429
24. Box**-**Ljung test
25. data**:** Squared.Residuals
26. X**-**squared **=** 0.19485, df **=** 1, p**-**value **=** 0.6589

**\*** GARCH Model Fit **\***

Conditional Variance Dynamics

**-----------------------------------**

GARCH Model **:** sGARCH**(**1,1**) #Standard GARCH with 1 ARCH and 1 GARCH**

Mean Model **:** ARFIMA**(**1,0,0**) Autoregressive Coefficient = 1**

Distribution **:** norm **Normal Distribution**

Optimal Parameters

**------------------------------------**

Estimate Std. Error t value Pr**(>|**t**|)**

mu 0.040058 0.023409 1.7112 0.087035 **Estimate: Not significant (< p value)**

ar1 0.195707 0.079685 2.4560 0.014049 **Estimate: Past values weakly influence current values (> p = 0.104)**

omega 0.027224 0.010352 2.6299 0.008542 **Estimate: Has a baseline volatility (constant term in GARCH)**

alpha1 0.231622 0.110327 2.0994 0.035780 **Estimate: Volatility spikes after large shocks (> p = 0.036)**

beta1 0.430026 0.164613 2.6123 0.008992 **Estimate: Volatility persistence (> p = 0.009)**

**For Standard Errors in general, conclusions remain similar, positive for heteroskedasticity. However, ‘mu’ turns insignificant (p = 0.23409 < 0.040058).**

**Because ‘alpha1’ and ‘beta1’ = 0.662 < 1, volatility must be ‘mean-reverting’ (a.k.a Price always returns back to their long-term mean or average).**

**However, because the ‘beta1’ value is considerably higher than the ‘alpha1’ value, the data seems to display the volatility shocks decaying slowly.**

Robust Standard Errors**:**

Estimate Std. Error t value Pr**(>|**t**|)**

mu 0.040058 0.026856 1.4916 0.135813

ar1 0.195707 0.081450 2.4028 0.016271

omega 0.027224 0.007189 3.7869 0.000153

alpha1 0.231622 0.099249 2.3337 0.019609

beta1 0.430026 0.106206 4.0490 0.000051

Log Likelihood **:** **-**23.11479

Information Criteria

**------------------------------------**

Akaike 0.28115

Bayes 0.36361

Shibata 0.27994

Hannan**-**Quinn 0.31452

Weighted Ljung**-**Box Test on Standardized Residuals

**------------------------------------**

statistic p**-**value

Lag**[**1**]** 0.1291 0.7194

Lag**[**2**\*(**p**+**q**)+(**p**+**q**)-**1**][**2**]** 0.3364 0.9912

Lag**[**4**\*(**p**+**q**)+(**p**+**q**)-**1**][**5**]** 2.5323 0.5572

d.o.f**=**1

H0 **:** No serial correlation **(No Autocorrelation) [Good Thing, due to all p-values being above 0.05, essentially capturing serial dependence].**

Weighted Ljung**-**Box Test on Standardized Squared Residuals

**------------------------------------**

statistic p**-**value

Lag**[**1**]** 0.5914 0.4419

Lag**[**2**\*(**p**+**q**)+(**p**+**q**)-**1**][**5**]** 2.1417 0.5853

Lag**[**4**\*(**p**+**q**)+(**p**+**q**)-**1**][**9**]** 3.7317 0.6346

d.o.f**=**2

**The GARCH(1, 1) model suffices and is good enough for these tests.**

Weighted ARCH LM Tests

**------------------------------------**

Statistic Shape Scale P**-**Value

ARCH Lag**[**3**]** 0.2981 0.500 2.000 0.5851

ARCH Lag**[**5**]** 0.7032 1.440 1.667 0.8225

ARCH Lag**[**7**]** 1.6444 2.315 1.543 0.7920

**No leftover volatility clustering as all P-Values are over 0.5**

**No ARCH effects at lags 3, 5, and 7.**

Nyblom stability test

**------------------------------------**

Joint Statistic**:** 0.5084

Individual Statistics**:**

mu 0.05495

ar1 0.23729

omega 0.08621

alpha1 0.12651

beta1 0.12095

Asymptotic Critical Values **(**10% 5% 1%)

Joint Statistic**:** 1.28 1.47 1.88

Individual Statistic**:** 0.35 0.47 0.75

**All characteristics’ statistics < critical values, therefore showing that the results portray no structural breaks.**

Sign Bias Test

**------------------------------------**

t**-**value prob sig

Sign Bias 2.3001 0.02250 **\*\***

Negative Sign Bias 0.9298 0.35360

Positive Sign Bias 0.3061 0.75983

Joint Effect 7.8229 0.04982 **\*\***

**Weak evidence of asymmetry (p = 0.049, compared to 0.05), could consider TGARCH if stronger.**

**From the sign bias, it’s evident that volatility reacts more to negative shocks.**

Adjusted Pearson Goodness**-**of**-**Fit Test**:**

**------------------------------------**

Group statistic p**-**value**(**g**-**1**)**

1 20 17.8 0.5358

2 30 34.9 0.2078

3 40 42.4 0.3266

4 50 57.5 0.1894

**As all P-Values are over 0.05, the normality assumption holds.**

**Overall, the GARCH(1, 1) model is well-specified with significant volatility effects, no real residual issues, whilst possessing stable parameters.**

**However, the only minor concern is potential asymmetry, however due to the nature of the financial stock markets, this could be nullified.**

**For further improvements you could implement a Student’s T-Distribution if residuals have fat tails, or other model variants such as TGARCH, or implement a second GARCH term (1, 2) to see if it improves fit.**