**Main Assumptions of Linear Regression Models:**

1. 𝗟 - 𝗟𝗶𝗻𝗲𝗮𝗿𝗶𝘁𝘆: The relationship between the independent and dependent variables should be linear.  
   If violated: The model may underperform, leading to biased predictions.
2. 𝗛 - 𝗛𝗼𝗺𝗼𝘀𝗰𝗲𝗱𝗮𝘀𝘁𝗶𝗰𝗶𝘁𝘆: The variance of residuals should be constant across all levels of the independent variable.  
   If violated: Leads to heteroscedasticity, making standard errors unreliable.
3. 𝗠 - 𝗠𝘂𝗹𝘁𝗶𝗰𝗼𝗹𝗹𝗶𝗻𝗲𝗮𝗿𝗶𝘁𝘆 (No Perfect Multicollinearity): Independent variables should not be highly correlated.  
   If violated: Model coefficients become unstable, affecting interpretation.
4. 𝗔 - 𝗔𝘂𝘁𝗼𝗰𝗼𝗿𝗿𝗲𝗹𝗮𝘁𝗶𝗼𝗻 (No Serial Correlation): Residuals should not be correlated.  
   If violated: Especially in time-series models, it can lead to inefficient predictions and misleading inferences.
5. 𝗡 - 𝗡𝗼𝗿𝗺𝗮𝗹𝗶𝘁𝘆 𝗼𝗳 𝗥𝗲𝘀𝗶𝗱𝘂𝗮𝗹𝘀: Residuals should be normally distributed (essential for small samples).  
   If violated: Hypothesis testing and confidence intervals may become unreliable.
6. Call**:**
7. garch**(**x **=** body\_mass\_change, order **=** c**(**1, 1**)**, trace **=** **FALSE)**
8. Model**:**
9. GARCH**(**1,1**)**
10. Residuals**:**
11. Min 1Q Median 3Q Max
12. **-**2.22070 **-**0.58309 0.05346 0.71924 2.90127
13. Coefficient**(**s**):**
14. Estimate Std. Error t value Pr**(>|**t**|)**
15. a0 0.02921 0.01466 1.992 0.0464 **\***
16. a1 0.20315 0.10562 1.923 0.0544 .
17. b1 0.44416 0.22592 1.966 0.0493 **\***
18. **---**
19. Signif. codes**:** 0 ‘**\*\*\***’ 0.001 ‘**\*\***’ 0.01 ‘**\***’ 0.05 ‘.’ 0.1 ‘ ’ 1
20. Diagnostic Tests**:**
21. Jarque Bera Test
22. data**:** Residuals
23. X**-**squared **=** 0.88359, df **=** 2, p**-**value **=** 0.6429
24. Box**-**Ljung test
25. data**:** Squared.Residuals
26. X**-**squared **=** 0.19485, df **=** 1, p**-**value **=** 0.6589

**\*** GARCH Model Fit **\***

Conditional Variance Dynamics

**-----------------------------------**

GARCH Model **:** sGARCH**(**1,1**) #Standard GARCH with 1 ARCH and 1 GARCH**

Mean Model **:** ARFIMA**(**1,0,0**) Autoregressive Coefficient = 1**

Distribution **:** norm **Normal Distribution**

Optimal Parameters

**------------------------------------**

Estimate Std. Error t value Pr**(>|**t**|)**

mu 0.040058 0.023409 1.7112 0.087035 **Estimate: Not significant (< p value)**

ar1 0.195707 0.079685 2.4560 0.014049 **Estimate: Past values weakly influence current values (> p = 0.104)**

omega 0.027224 0.010352 2.6299 0.008542 **Estimate: Has a baseline volatility (constant term in GARCH)**

alpha1 0.231622 0.110327 2.0994 0.035780 **Estimate: Volatility spikes after large shocks (> p = 0.036)**

beta1 0.430026 0.164613 2.6123 0.008992 **Estimate: Volatility persistence (> p = 0.009)**

**For Standard Errors in general, conclusions remain similar, positive for heteroskedasticity. However, ‘mu’ turns insignificant (p = 0.23409 < 0.040058).**

**Because ‘alpha1’ and ‘beta1’ = 0.662 < 1, volatility must be ‘mean-reverting’ (a.k.a Price always returns back to their long-term mean or average).**

**However, because the ‘beta1’ value is considerably higher than the ‘alpha1’ value, the data seems to display the volatility shocks decaying slowly.**

Robust Standard Errors**:**

Estimate Std. Error t value Pr**(>|**t**|)**

mu 0.040058 0.026856 1.4916 0.135813

ar1 0.195707 0.081450 2.4028 0.016271

omega 0.027224 0.007189 3.7869 0.000153

alpha1 0.231622 0.099249 2.3337 0.019609

beta1 0.430026 0.106206 4.0490 0.000051

Log Likelihood **:** **-**23.11479

Information Criteria

**------------------------------------**

Akaike 0.28115

Bayes 0.36361

Shibata 0.27994

Hannan**-**Quinn 0.31452

Weighted Ljung**-**Box Test on Standardized Residuals

**------------------------------------**

statistic p**-**value

Lag**[**1**]** 0.1291 0.7194

Lag**[**2**\*(**p**+**q**)+(**p**+**q**)-**1**][**2**]** 0.3364 0.9912

Lag**[**4**\*(**p**+**q**)+(**p**+**q**)-**1**][**5**]** 2.5323 0.5572

d.o.f**=**1

H0 **:** No serial correlation **(No Autocorrelation) [Good Thing, due to all p-values being above 0.05, essentially capturing serial dependence].**

Weighted Ljung**-**Box Test on Standardized Squared Residuals

**------------------------------------**

statistic p**-**value

Lag**[**1**]** 0.5914 0.4419

Lag**[**2**\*(**p**+**q**)+(**p**+**q**)-**1**][**5**]** 2.1417 0.5853

Lag**[**4**\*(**p**+**q**)+(**p**+**q**)-**1**][**9**]** 3.7317 0.6346

d.o.f**=**2

**The GARCH(1, 1) model suffices and is good enough for these tests.**

Weighted ARCH LM Tests

**------------------------------------**

Statistic Shape Scale P**-**Value

ARCH Lag**[**3**]** 0.2981 0.500 2.000 0.5851

ARCH Lag**[**5**]** 0.7032 1.440 1.667 0.8225

ARCH Lag**[**7**]** 1.6444 2.315 1.543 0.7920

**No leftover volatility clustering as all P-Values are over 0.5**

**No ARCH effects at lags 3, 5, and 7.**

Nyblom stability test

**------------------------------------**

Joint Statistic**:** 0.5084

Individual Statistics**:**

mu 0.05495

ar1 0.23729

omega 0.08621

alpha1 0.12651

beta1 0.12095

Asymptotic Critical Values **(**10% 5% 1%)

Joint Statistic**:** 1.28 1.47 1.88

Individual Statistic**:** 0.35 0.47 0.75

**All characteristics’ statistics < critical values, therefore showing that the results portray no structural breaks.**

Sign Bias Test

**------------------------------------**

t**-**value prob sig

Sign Bias 2.3001 0.02250 **\*\***

Negative Sign Bias 0.9298 0.35360

Positive Sign Bias 0.3061 0.75983

Joint Effect 7.8229 0.04982 **\*\***

**Weak evidence of asymmetry (p = 0.049, compared to 0.05), could consider TGARCH if stronger.**

**From the sign bias, it’s evident that volatility reacts more to negative shocks.**

Adjusted Pearson Goodness**-**of**-**Fit Test**:**

**------------------------------------**

Group statistic p**-**value**(**g**-**1**)**

1 20 17.8 0.5358

2 30 34.9 0.2078

3 40 42.4 0.3266

4 50 57.5 0.1894

**As all P-Values are over 0.05, the normality assumption holds.**

**Overall, the GARCH(1, 1) model is well-specified with significant volatility effects, no real residual issues, whilst possessing stable parameters.**

**However, the only minor concern is potential asymmetry, however due to the nature of the financial stock markets, this could be nullified.**

**For further improvements you could implement a Student’s T-Distribution if residuals have fat tails, or other model variants such as TGARCH, or implement a second GARCH term (1, 2) to see if it improves fit.**

Estimate Std. Error t value Pr**(>|**t**|)**

omega 0.022067 0.001433 15.3976 0.000000 Baseline log-volatility, which is small, positive, yet significant (as P-Value is equal to 0).

alpha1 **-**0.070481 0.013570 **-**5.1940 0.000000 Impact of past shocks on volatility are negative, yet also very significant, which indicates that the volatility reacts to past returns.

beta1 0.985439 0.003912 251.9247 0.000000 Represents the persistence of volatility, of which is also very high and significant, suggesting a long memory in volatility.

gamma1 0.176245 0.076490 2.3042 0.021214 Asymmetry (a.k.a., the leverage effect), being positive and significant (as p = 0.021214), meaning negative shocks are bound to increase volatility more than positive shocks.

skew 0.986317 0.025605 38.5210 0.000000 Skewness parameter, indicating a slight negative skew (more downside risk for BOEING returns).

shape 4.331630 0.352964 12.2722 0.000000 Degrees of freedom for T-Dist., the low value of ‘4’ essentially confirms the presence of ‘fat tails’.

Ultimately, all parameters are statistically significant (p < 0.05) under regular standard errors given, so unlike the analyst’s normal distribution, the skewed t-distribution actually captures more of the data (fat tails) as well as the asymmetry, addressing the excessive VaR exceedances (26 vs 10).

Overall, some p-values increase (e.g, gamma1 = p = 0.557), however the core parameters such as the alpha, beta, skew, and shape, remain significant, therefore suggesting ‘robustness’.

(No autocorrelation is a good feature, and an assumption for all (or at least most) valid econometrics models)

Tests for remaining volatility clustering (H0: no autocorrelation in squared residuals). All p-values > 0.05, so the EGARCH(1,1) effectively captures volatility dynamics. This is a big improvement over the analyst’s model, where the CC test rejection suggested clustering issues.

Tests if positive/negative shocks asymmetrically affect volatility beyond EGARCH’s modelling (H0: no bias).

All p-values > 0.05, so H0 is (finally) not rejected. The gamma1 parameter (0.176) adequately captures the leverage effect, therefore improving on the analyst’s GARCH(1, 1) model, which was shown to have mis-specified asymmetry.